# USER'S MANUAL FOR THREE DIMENSIONAL FDTD VERSION D CODE FOR SCATTERING FROM FREQUENCY-DEPENDENT DIELECTRIC AND MAGNETIC MATERIALS

by

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### I. INTRODUCTION

The Penn State Finite Difference Time Domain Electromagnetic Scattering Code Version D is a three dimensional numerical electromagnetic scattering code based upon the Finite Difference Time Domain Technique (FDTD). The supplied version of the code is one version of our current three dimensional FDTD code set. This manual provides a description of the code and corresponding The manual is organized results for several scattering problems. introduction, description of the FDTD into fourteen sections: method, operation, resource requirements, Version D code capabilities, a brief description of the default scattering geometry, a brief description of each subroutine, a description of the include file (COMMOND.FOR), a section briefly discussing Radar Cross Section (RCS) computations, a section discussing some scattering results, a sample problem setup section, a new problem checklist, references and figure titles.

# II. FDTD METHOD

The Finite Difference Time Domain (FDTD) technique models transient electromagnetic scattering and interactions with objects of arbitrary shape and/or material composition. The technique was first proposed by Yee [1] for isotropic, non-dispersive materials in 1966; and has matured within the past twenty years into a robust and efficient computational method. The present FDTD technique is capabable of transient electromagnetic interactions with objects of arbitrary and complicated geometrical shape and material composition over a large band of frequencies. This technique has recently been extended to include dispersive dielectric materials, chiral materials and plasmas.

In the FDTD method, Maxwell's curl equations are discretized in time and space and all derivatives (temporal and spatial) are approximated by central differences. The electric and magnetic fields are interleaved in space and time and are updated in a second-order accurate leapfrog scheme. The computational space is divided into cells with the electric fields located on the edges and the magnetic fields on the faces (see Figure 1). FDTD objects are defined by specifying dielectric and/or magnetic material parameters at electric and/or magnetic field locations.

Two basic implementations of the FDTD method are widely used for electromagnetic analysis: total field formalism and scattered field formalism. In the total field formalism, the electric and magnetic field are updated based upon the material type present at each spatial location. In the scattered field formalism, the incident waveform is defined analytically and the scattered field is coupled to the incident field through the different material types. For the incident field, any waveform, angle of incidence and polarization is possible. The separation of the incident and

scattered fields conveniently allows an absorbing boundary to be employed at the extremities of the discretized problem space to absorb the scattered fields.

This code is a scattered field code, and the total E and H fields may be found by combining the incident and scattered fields. Any type of field quantity (incident, scattered, or total), Poynting vector or current are available anywhere within the computational space. These fields, incident, scattered and total, may be found within, on or about the interaction object placed in the problem space. By using a near to far field transformation, far fields can be determined from the near fields within the problem space thereby affording radiation patterns and RCS values. The accuracy of these calculations is typically within a dB of analytic solutions for dielectric and magnetic sphere scattering. Further improvements are expected as better absorbing boundary conditions are developed and incorporated.

# III. OPERATION

Typically, a truncated Gaussian incident waveform is used to excite the system being modeled, however certain code versions also provide a smooth cosine waveform for convenience in modeling dispersive materials. The interaction object is defined in the discretized problem space with arrays at each cell location created by the discretization. All three dielectric material types for E field components within a cell can be individually specified by the arrays IDONE(I,J,K), IDTWO(I,J,K), IDTHRE(I,J,K). This models arbitrary dielectric materials with  $\mu = \mu_0$ . By an obvious extension to six arrays, magnetic materials with  $\mu \neq \mu_0$  can be modeled.

Scattering occurs when the incident wave, marched forward in time in small steps set by the Courant stability condition, reaches the interaction object. Here a scattered wave must appear along with the incident wave so that the Maxwell equations are satisfied. If the material is a perfectly conductive metal then only the well known boundary condition

$$E_{tan}^{scat} = -E_{tan}^{inc} \tag{1}$$

must be satisfied. For a nondispersive dielectric the requirement is that the total field must satisfy the Maxwell equations in the material:

$$\nabla \times \mathbf{E}^{\text{tot}} = \nabla \times (\mathbf{E}^{\text{inc}} + \mathbf{E}^{\text{scat}}) = -\frac{1}{\mu_{o}} \frac{\partial \mathbf{H}^{\text{tot}}}{\partial \mathbf{t}} = -\frac{1}{\mu_{o}} \frac{\partial (\mathbf{H}^{\text{inc}} + \mathbf{H}^{\text{scat}})}{\partial \mathbf{t}}$$
(2)

$$\nabla \times \mathbf{H}^{\text{tot}} = \nabla \times (\mathbf{H}^{\text{inc}} + \mathbf{H}^{\text{scat}}) = \varepsilon \frac{\partial \mathbf{E}^{\text{tot}}}{\partial \mathbf{t}} + \sigma \mathbf{E}^{\text{tot}}$$
(3)

$$= \varepsilon \frac{\partial (E^{inc} + E^{scat})}{\partial t} + \sigma (E^{inc} + E^{scat})$$
 (4)

Additionally the incident wave, defined as moving unimpeded through a vacuum in the problem space, satisfies everywhere in the problem the Maxwell equations for free space

$$\nabla \times \mathbf{E}^{inc} = -\frac{1}{\mu_o} \frac{\partial \mathbf{H}^{inc}}{\partial \mathbf{t}}$$
 (5)

$$\nabla \times \mathbf{H}^{\text{inc}} = \varepsilon_{0} \frac{\partial \mathbf{E}^{\text{inc}}}{\partial \mathbf{t}} \tag{6}$$

Subtracting the second set of equations from the first yields the Maxwell equations governing the scattered fields in the material:

$$\nabla \times \mathbf{E}^{\text{scat}} = -\frac{1}{\mu_0} \frac{\partial \mathbf{H}^{\text{scat}}}{\partial \mathbf{t}} \tag{7}$$

$$\nabla \times \mathbf{H}^{\text{scat}} = (\varepsilon - \varepsilon_{\text{o}}) \frac{\partial \mathbf{E}^{\text{inc}}}{\partial \mathbf{t}} + \sigma \mathbf{E}^{\text{inc}} + \varepsilon \frac{\partial \mathbf{E}^{\text{scat}}}{\partial \mathbf{t}} + \sigma \mathbf{E}^{\text{scat}}$$
(8)

Outside the material this simplifies to:

$$\nabla \times \mathbf{E}^{\text{scat}} = -\frac{1}{\mu_0} \frac{\partial \mathbf{H}^{\text{scat}}}{\partial \mathbf{t}} \tag{9}$$

$$\nabla \times \mathbf{H}^{\text{scat}} = \varepsilon_0 \frac{\partial \mathbf{E}^{\text{scat}}}{\partial \mathbf{t}}$$
 (10)

Magnetic materials, dispersive effects, non-linearities, etc., are further generalizations of the above approach. Based on the value of the material type, the subroutines for calculating scattered E and H field components branch to the appropriate expression for that scattered field component and

that component is advanced in time according to the selected algorithm. As many materials can be modeled as desired, the number equals the dimension selected for the flags. If materials with behavior different from those described above must be modeled, then after the appropriate algorithm is found, the code's branching structure allows easy incorporation of the new behavior.

# IV. RESOURCE REQUIREMENTS

The number of cells the problem space is divided into times the six components per cell set the problem space storage requirements

Storage = 
$$NC \times 6$$
 components/cell  $\times 4$  bytes/component (11)

and the computational cost

Operations = 
$$NC \times 6 \text{ comp/cell} \times 10 \text{ ops/component} \times N$$
 (12)

where N is the number of time steps desired.

N typically is on the order of ten times the number of cells on one side of the problem space. More precisely for cubical cells it takes  $\sqrt{3}$  time steps to traverse a single cell when the time step is set by the Courant stability condition

$$\Delta t = \frac{\Delta x}{\sqrt{3}c} \qquad \Delta x = cell size dimension$$
 (13)

The condition on N is then that

$$N = 10 \times (\sqrt{3} NC^{\frac{1}{3}})$$
  $NC^{\frac{1}{3}} \sim \text{number cells on a side}$  (14)

The earliest aircraft modeling using FDTD with approximately 30 cells on a side required approximately 500 time steps. For more recent modeling with approximately 100 cells on a side, 2000 or more time steps are used.

For (100 cell)<sup>3</sup> problem spaces, 24 MBytes of memory are required to store the fields. Problems on the order of this size have been run on a Silicon Graphics 4D 220 with 32 MBytes of memory, IBM RISC 6000, an Intel 486 based machine, and VAX 11/785. Storage is only a problem as in the case of the 486 where only 16 MBytes of memory was available. This limited the problem space size to approximately (80 cells)<sup>3</sup>.

For  $(100 \text{ cell})^3$  problems with approximately 2000 time steps, there is a total of 120 x 10° operations to perform. The speeds of the previously mentioned machines are 24 MFLOPS (4 processor upgraded version), 10 MFLOPS, 1.5 MFLOPS, and 0.2 MFLOPS. The run times are then  $5 \times 10^3$  seconds, 12 x  $10^3$  seconds, 80 x  $10^3$  seconds and  $600 \times 10^3$  seconds, respectively. In hours the times are 1.4, 3.3, 22.2 and 167 hours. Problems of this size are possible on all but the last machine and can in fact be performed on a personal computer (486) if one day turnarounds are permissible.

# V. VERSION D CODE CAPABILITIES

The Penn State University FDTD Electromagnetic Scattering Code Version D has the following capabilities:

- 1) Ability to model lossy dielectric and perfectly conducting scatterers.
- 2) Ability to model lossy magnetic scatterers.
- 3) Ability to model dispersive dielectric and dispersive magnetic scatterers. This dispersive FDTD method is now designated (FD) TD for Frequency-Dependent Finite Difference Time Domain.
- 4) First and second order outer radiation boundary condition (ORBC) operating on the electric fields for dielectric and dispersive dielectric scatterers.
- 5) First and second order ORBC operating on the magnetic fields for magnetic and dispersive magnetic scatterers.
- 6) Near to far zone transformation capability to obtain far zone scattered fields.
- 7) Gaussian and smooth cosine incident waveforms with arbitrary incidence angles.
- 8) Near zone field, current or power sampling capability.
- 9) Companion code for computing Radar Cross Section (RCS).

### VI. DEFAULT SCATTERING GEOMETRY

The code as delivered is set up to calculate the far zone backscatter fields for a 6.67 meter radius, dispersive, Nickel Ferrite sphere. Nickel Ferrite is defined by a frequency dependent permeability given by

where  $\mu_{\mathbf{w}}$  is the infinite frequency permeability,  $\mu_{\mathbf{s}}$  is the zero

$$\frac{\mu(\omega)}{\mu_0} = \mu_{\infty} + \frac{\mu_{s} - \mu_{\infty}}{1 + j\omega\tau_0}$$
 (15)

frequency permeability,  $\tau_0$  is the relaxation time and  $\omega$  is the radian frequency. The Nickel Ferrite parameters are  $\mu_{\omega}=1$ ,  $\mu_{\rm s}=100$  and  $\tau_0=20$  ns. The problem space size is 66 by 66 by 66 cells in the x, y and z directions, the cells are 1/3 m cubes, and the incident waveform is a  $\phi$ -polarized smooth cosine pulse with incidence angles of  $\theta=22.5$  and  $\phi=22.5$  degrees. The output data files are included as a reference along with a code (RCS3D.FOR) for computing the frequency domain RCS using these output data files. The ORBC is the second order absorbing boundary condition set forth by Mur [2].

# VII. SUBROUTINE DESCRIPTION

In the description for each subroutine, an asterisk (\*) will be placed by the subroutine name if that particular subroutine is normally modified when defining a scattering problem.

# MAIN ROUTINE

The main routine in the program contains the calls for all necessary subroutines to initialize the problem space and scattering object(s) and for the incident waveform, far zone transformation, field update subroutines, outer radiation boundary conditions and field sampling.

The main routine begins with the include statement and then appropriate data files are opened, and subroutines ZERO, BUILD and SETUP are called to initialize variables and/or arrays, build the object(s) and initialize the incident waveform and miscellaneous parameters, respectively. Subroutine SETFZ is called to intialize parameters for the near to far zone transformation if far zone fields are desired.

The main loop is entered next, where all of the primary field computations and data saving takes place. During each time step cycle, the EXSFLD, EYSFLD, and EZSFLD subroutines are called to update the x, y, and z components of the scattered electric field. The six electric field outer radiation boundary conditions (RADE??) are called next to absorb any outgoing scattered fields for perfectly conducting, dielectric, or dispersive dielectric scatterers. Time is then advanced 1/2 time step according to the Yee algorithm and then the HXSFLD, HYSFLD, AND HZSFLD subroutines are called to update the x, y, and z components of scattered magnetic field. The six magnetic field

outer radiation boundary conditions (RADH??) are called next to absorb any outgoing scattered fields for magnetic or dispersive magnetic scatterers. Time is then advanced another 1/2 step and then either near zone fields are sampled and written to disk in DATSAV, and/or the near zone to far zone vector potentials are updated in SAVFZ. The parameter NZFZ (described later) in the common file defines the type of output fields desired.

After execution of all time steps in the main field update loop, subroutine FAROUT is called if far zone fields are desired to compute the far zone fields and write them to disk. At this point, the execution is complete.

# SUBROUTINE SETFZ

This subroutine initializes the necessary parameters required for far zone field computations. The code as furnished computes backscatter far zone fields and can compute bistatic far zone fields for one scattering angle (i.e. one  $\theta$  and  $\phi$  angle). Refer to reference [3] for a complete description of the near to far zone transformation. Other versions of this subroutine provide for multiple bistatic angles.

# SUBROUTINE SAVFZ

This subroutine updates the near zone to far zone vector potentials.

# SUBROUTINE FAROUT

This subroutine changes the near zone to far zone vector potentials to far zone electric field  $\theta$  and  $\phi$  components and writes them to disk.

# SUBROUTINE BUILD \*

This subroutine "builds" the scattering object(s) by initializing the IDONE, IDTWO, IDTHRE, IDFOR, IDFIV and IDSIX The IDONE-IDTHRE arrays are for specifying perfectly conducting, lossy dielectrics and dispersive dielectric The IDFOR-IDSIX arrays are for lossy magnetic and materials. dispersive magnetic materials. The reason for the separate arrays is so the user can independently control the exact placement of dielectric and magnetic material in the Yee cells. Refer to Figure 1 for a diagram of the basic Yee cell. example, setting an element of the IDONE array at some I, J, K location is actually locating dielectric material at a cell edge whose center location is I+0.5, J, K. Setting an element of the IDFOR array at some I, J, K location is actually locating magnetic material perpendicular to a cell face whose center location is I,J+0.5,K+0.5, or equivalently, locating magnetic material at an edge on the dual H field mesh. The difference between the IDONE and IDFOR array locations is a direct result of the field offsets in the Yee cell (see Figure 1). Thus, materials with diagonal permittivity and/or diagonal permeability tensors can be modeled. The default material type for all ID??? arrays is 0, or free By initializing these arrays to values other than 0, the user is defining an object by determining what material types are present at each spatial location. Other material types available for IDONE-IDTHRE are 1 for perfectly conducting objects, 2-9 for lossy non-magnetic dielectrics, 20-29 for dispersive dielectrics. IDONE-IDTHRE are normally set to 0 for magnetic scatterers. Other material types available for IDFOR-IDSIX are 10-19 for lossy magnetic materials and 30-39 for dispersive magnetic materials. IDFOR-IDSIX are normally set to 0 for perfectly conducting or dielectric scatterers. If the user wants a material with both dielectric and magnetic properties (i.e. permittivity other than  $\epsilon_0$  for magnetic materials, and permeability other than  $\mu_0$  for dielectric materials), then he/she must define IDONE-IDSIX for that particular material. This subroutine also has a section that checks the ID??? arrays to determine if legal material types have been defined throughout the problem space. The actual nondispersive material parameters ( $\epsilon$ ,  $\mu$ ,  $\sigma$ , and  $\sigma$ ) are defined in subroutine SETUP. The dispersive material parameters ( $\epsilon_{\rm s}$ ,  $\epsilon_{\rm m}$ ,  $\tau_{\rm 0}$  ,  $\sigma$  ,  $\mu_{\rm s}$  ,  $\mu_{\rm m}$  ,  $\tau_{\rm 0}$  , and  $\sigma^{\rm *}$  ) are also defined in a separate section

The user must be careful that his/her object created in the BUILD subroutine is properly formed. There is not a direct one-to-one correspondence between the dielectric and magnetic ID??? arrays. However, one can define a correspondence, so that code used to generate a dielectric object can be modified to generate a magnetic object.

in SETUP. The default geometry is a 6.67 m radius, dispersive,

To see this consider that we have set the permittivity at cell locations corresponding to

$$EX(I,J,K)$$
,  $EY(I,J,K)$ ,  $EZ(I,J,K)$ 

Nickel Ferrite sphere.

using the IDONE, IDTWO, and IDTHRE arrays respectively. This determines one corner of a dielectric cube. If we wish to define the corner of a corresponding magnetic cube, offset 1/2 cell in the x, y, z directions, we would set the locations of the magnetic fields

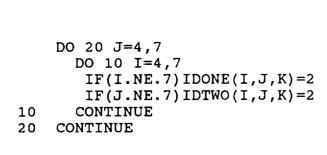
$$HX(I+1,J,K)$$
,  $HY(I,J+1,K)$ ,  $HZ(I,J,K+1)$ 

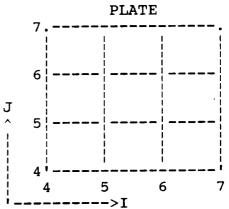
as magnetic material using the IDFOR, IDFIV, and IDSIX arrays.

This example indicates the following correspondence between BUILDing dielectric and magnetic objects:

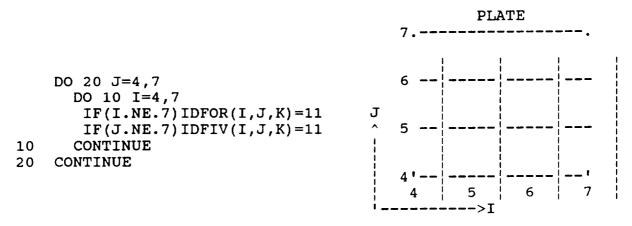
Dielectric Object		Magnetic Object
IDONE(I,J,K)	=	IDFOR(I+1,J,K)
IDTWO(I,J,K)	=	IDFIV(I,J+1,K)
IDTHRE(I,J,K)	=	IDSIX(I,J,K+1)

To illustrate this for a somewhat more complicated case, consider an example of a 3x3 cell dielectric plate located in the XY plane at K=5. The plate can be generated with the following FORTRAN lines:





If the same FORTRAN lines were used to try to generate a magnetic plate, the object generated would actually be unconnected:



The correct way to build the magnetic plate is with the following FORTRAN code:

This is equivalent to the correspondence relationship given above. See comments in the BUILD subroutine for further explanation of the ID??? arrays.

When it is important to place the object in the center of the problem space (to have lowest possible cross-pol scattering for symmetric objects), NX etc. should be odd for dielectrics and even for magnetics. This is due to the field locations in the Yee cell and also the placement of the E and H field absorbing boundary condition surfaces.

If the object being modeled has curved surfaces, edges, etc. that are at an angle to one or more of the coordinate axes, then that shape must be approximately modeled by lines and faces in a "stair-stepped" (or stair-cased) fashion. This stair-cased approximation introduces errors into computations at higher frequencies. Intuitively, the error becomes smaller as more cells are used to stair-case a particular object. Thus, the default Nickel Ferrite sphere scattering geometry is a stair-cased sphere.

When the user's test object is dielectric it is apparently better to use the Mur E field boundary formulation. For a magnetic scattering object it seems better to use the Mur H field boundary formulation. The reason for this has to do with the reactive part of the energy (near zone fields) reaching the absorbing boundary surface. These near zone fields differ for magnetic and dielectric objects, even for dual objects where the far fields are essentially identical.

One final comment on building dual dielectric vs magnetic objects can be explained by considering the duality between magnetic and dielectric materials. In testing the code, it was helpful to test dual dielectric/magnetic objects since they scatter identically in the far zone. In specifying the material of a magnetic scatterer to be the dual of a dielectric, the following transformation must be applied:

DIELECTRIC	<dual></dual>	MAGNETIC
E field	<>	H field
H field	<>	-E field
$\epsilon$	<>	$\mu$
μ	<>	$\epsilon$
ß (k)	<>	ß (k) *
σ	<>	$\sigma \cdot (\mu_0 / \epsilon_0) = \sigma$

The somewhat surprising entry in this table is the relationship between the dielectric and magnetic conductivities. The reason why the magnetic conductivity,  $\sigma$ , for magnetic materials has to be scaled as indicated is that for duality to be applied the free

space impedance must be inverted. Although this is not generally possible for actual problems, identical far zone scattering for dielectric and magnetic scatterers can be achieved by this scaling of  $\sigma$  as above and realizing that E and H scattered field to incident field ratios will not invert. This scaling of  $\sigma$  is why conductivity for magnetic materials is usually not a dominating feature, and in fact is often neglected.

### SUBROUTINE DCUBE

This subroutine builds cubes of dielectric material by defining four each of IDONE, IDTWO and IDTHRE components corresponding to one spatial cube of dielectric material. It can also be used to define thin (i.e. up to one cell thick) dielectric or perfectly conducting plates. Refer to comments within DCUBE for a description of the arguments and usage of the subroutine. This subroutine could be modified to build cubes and/or plates of magnetic materials by using a triple do-loop in BUILD (after calls to DCUBE) over coordinate indices I, J, K and applying the correspondence between the IDONE-IDTHRE and IDFOR-IDSIX arrays that was previously discussed.

# SUBROUTINE SETUP \*

This subroutine initializes many of the constants required for incident field definition, field update equations, outer radiation boundary conditions and material parameters. The material parameters  $\epsilon$ ,  $\mu$ ,  $\sigma$  and  $\sigma$  are defined for each material type (non-dispersive) using the material arrays EPS, XMU, SIGMA and SIGMAC respectively. The array EPS is used for the total permittivity, XMU is used for the total permeability, SIGMA is used for the electric conductivity and SIGMAC is used for the magnetic conductivity (useful for running dual problems). These arrays are initialized in SETUP to free space material parameters for all material types and then the user is required to modify these arrays for his/her scattering materials. Thus, for the lossy dielectric material type 2, the user must define EPS(2) and SIGMA(2).

For dispersive dielectric and magnetic materials, different material parameter arrays are used. The functional form of the frequency dependent permittivity/permeability that was implemented in the code is the Debye relaxation [4] with an effective DC conductivity given by

$$\epsilon(\omega) = \epsilon' - j\epsilon'' = \epsilon_{\omega}\epsilon_0 + \epsilon_0\chi_e(\omega) + \frac{\sigma}{j\omega}$$
 (16)

where the frequency dependent electric susceptibility function is defined as

$$\chi_{e}(\omega) = \frac{\epsilon_{s} - \epsilon_{\infty}}{1 + j\omega\tau_{0}}$$
 (17)

where  $\epsilon_{\infty}$  is the infinite frequency permittivity,  $\epsilon_{\rm S}$  is the zero frequency permittivity,  $\tau_0$  is the relaxation time,  $\sigma$  is the effective electric conductivity, and  $\omega$  is the radian frequency. The same expressions were used for the frequency dependent permeability and can be written from equations (16) and (17) by replacing  $\epsilon$  by  $\mu$  and  $\sigma$  by  $\sigma$ . The corresponding time domain susceptibility function is given by

$$\chi_{e}(t) = \left(\frac{\epsilon_{s} - \epsilon_{\varpi}}{\tau_{0}}\right) e^{\left(-t/\tau_{o}\right)} u(t)$$
 (18)

The FDTD implementation of frequency dependent permittivity and/or permeability involves a convolution with the electric and/or magnetic field and interested readers are referred to references [5-6] for further details.

For dispersive dielectric materials, the corresponding material parameter arrays are EPSINF ( $\epsilon_{\rm w}$ ), EPSSTA ( $\epsilon_{\rm s}$ ), RELAXT ( $\tau_{\rm 0}$ ), and RELSIG ( $\sigma$ ). For dispersive magnetic materials, the arrays are XMUINF ( $\mu_{\rm w}$ ), XMUSTA ( $\mu_{\rm s}$ ), RELAXT ( $\tau_{\rm 0}$ ) and RELSIG ( $\sigma$ ). These dispersive material parameters are defined under the DISPERSIVE SETUP portion of the subroutine. The remainder of the subroutine computes constants used in field update equations and boundary conditions and writes the diagnostics file.

# SUBROUTINE EXSFLD

This subroutine updates all x components of scattered electric field at each time step except those on the outer boundaries of the problem space. If statements based upon the IDONE array are used to determine the type of material present and the corresponding update equation to be used. These scattered field equations are based on the development given in [7].

### SUBROUTINE EYSFLD

This subroutine updates all y components of scattered electric field at each time step except those on the outer boundaries of the problem space. IF statements based upon the IDTWO array are used to determine the type of material present

and the corresponding update equation to be used.

# SUBROUTINE EZSFLD

This subroutine updates all z components of scattered electric field at each time step except those on the outer boundaries of the problem space. If statements based upon the IDTHRE array are used to determine the type of material present and the corresponding update equation to be used.

SUBROUTINES RADEYX, RADEZX, RADEZY, RADEXY, RADEXZ and RADEYZ

These subroutines apply the outer radiation boundary conditions to the scattered electric field on the outer boundaries of the problem space for non-magnetic scatterers. The user controls selection of these routines through the parameter MAGNET (described later).

### SUBROUTINE HXSFLD

This subroutine updates all x components of scattered magnetic field at each time step. IF statements based upon the IDFOR array are used to determine the type of material present and the corresponding update equation to be used.

### SUBROUTINE HYSFLD

This subroutine updates all y components of scattered magnetic field at each time step. IF statements based upon the IDFIV array are used to determine the type of material present and the corresponding update equation to be used.

# SUBROUTINE HZSFLD

This subroutine updates all z components of scattered magnetic field at each time step. IF statements based upon the IDSIX array are used to determine the type of material present and the corresponding update equation to be used.

SUBROUTINES RADHXZ, RADHXX, RADHXY, RADHXY, RADHXZ and RADHXX

These subroutines apply the outer radiation boundary conditions to the scattered magnetic field on the outer boundaries of the problem space for magnetic scatterers. The user controls selection of these routines through the parameter MAGNET (described later).

This subroutine samples near zone scattered field quantities and saves them to disk. This subroutine is where the quantities to be sampled and their spatial locations are to be specified and is only called if near zone fields only are desired or if both near and far zone fields are desired. Total field quantities can also be sampled. See comments within the subroutine for specifying sampled scattered and/or total field quantities. When sampling magnetic fields, remember the  $\delta t/2$  time difference between E and H when writing the fields to disk. Sections of code within this subroutine determine if the sampled quantities and the spatial locations have been properly defined.

# FUNCTIONS EXI, EYI, EZI, HXI, HYI and HZI

These functions are called to compute the x, y and z components of incident electric and magnetic field, respectively. The functional form of the incident field is contained in a separate function SOURCE.

### **FUNCTION SOURCE \***

This function contains the functional form of the incident field. The code as furnished uses the smooth cosine form of the incident field. An incident Gaussian pulse is also available by uncommenting the required lines and commenting out the smooth cosine pulse. Thus, this function need only be modified if the user changes the incident pulse from smooth cosine to Gaussian. Currently, only the smooth cosine pulse can be used for scattering from dispersive targets. A slight improvement in computing speed and vectorization may be achieved by moving this function inside each of the incident field functions EXI, EYI and so on.

# FUNCTIONS DEXI, DEYI, DEZI, DHXI, DHYI and DHZI

These functions are called to compute the x, y and z components of the time derivative of incident electric and magnetic field, respectively. The functional form of the incident field is contained in a separate function DSRCE.

FUNCTIONS DEXIXE, DEYIXE, DEZIXE, DHXIXE, DHYIXE and DHZIXE

These functions compute the x, y and z components of the convolution of the time derivative of the incident field with the electric or magnetic Debye susceptibility function  $\chi$ .

This function contains the functional form of the time derivative of the incident field. The code as furnished uses the time derivative of the smooth cosine form of the incident field. A Gaussian pulse time derivative is also available by uncommenting the required lines and commenting out the smooth cosine pulse. Thus, the function need only be modified if the user changes from the smooth cosine to Gaussian pulse. Again, a slight improvement in computing speed and vectorization may be achieved by moving this function inside each of the time derivative incident field functions DEXI, DEYI and so on.

### FUNCTION DCONV

This function evaluates the convolution of the time derivative of the incident field with the Debye susceptibility function  $\chi$ .

### SUBROUTINE ZERO

This subroutine initializes various arrays and variables to zero.

# VIII. INCLUDE FILE DESCRIPTION (COMMOND.FOR) \*

The include file, COMMOND.FOR, contains all of the arrays and variables that are shared among the different subroutines. This file will require the most modifications when defining scattering problems. A description of the parameters that are normally modified follows.

The parameters NX, NY and NZ specify the size of the problem space in cells in the x, y and z directions respectively. For problems where it is crucial to center the object within the problem space, then NX, NY and NZ should be odd for dielectric scatterers and even for magnetic scatterers. The parameter NTEST defines the number of near zone quantities to be sampled and NZFZ defines the field output format. Set NZFZ=0 for near zone fields only, NZFZ=1 for far zone fields only and NZFZ=2 for both near and far zone fields. Parameter MAGNET is used to define magnetic scatterers and it controls the choice of RADE?? versus RADH?? absorbing boundary subroutines. It is set to 0 for dielectric scatterers and to 1 for magnetic scatterers. Parameter NUMMAT defines the total number of material types that are available for NEDISP and NHDISP define the number of dispersive dielectric and magnetic materials that are being used. Parameter NSTOP defines the maximum number of time steps. DELX, DELY, and DELZ (in meters) define the cell size in the x, y and z directions respectively. The heta and  $\phi$  incidence angles (in degrees) are defined by THINC and PHINC respectively and the polarization is defined by ETHINC and EPHINC. ETHINC=1.0, EPHINC=0.0 for  $\theta$ -polarized incident field and ETHINC=0.0,

EPHINC=1.0 for  $\phi$ -polarized incident fields. Parameters AMP and BETA define the maximum amplitude and the e² temporal width of the incident pulse respectively. BETA automatically adjusts when the cell size is changed and normally should not be changed by the user. The far zone scattering angles are defined by THETFZ and PHIFZ. The code as furnished performs backscatter computations, but these parameters could be modified for a bistatic computation.

# IX. RCS COMPUTATIONS

A companion code, RCS3D.FOR, has been included to compute RCS versus frequency. It uses the file name of the (FD) TD far zone output data (FZOUT3D.DAT) and writes data files of far zone electric fields versus time (FZTIME.DAT) and RCS versus frequency (3DRCS.DAT). The RCS computations are performed up to the 10 cell/ $\lambda_0$  frequency limit. Refer to comments within this code for further details.

## X. RESULTS

As previously mentioned, the code as furnished models a 6.67 m radius, dispersive, Nickel Ferrite sphere and computes backscatter far zone scattered fields at angles of  $\theta$ =22.5 and  $\phi$ =22.5 degrees. Results are included for 0.25 dB loaded dielectric and magnetic foam spheres, 60 dB dielectric and magnetic foam spheres and the Nickel Ferrite magnetic sphere. The material parameters for 0.25 dB and 60 dB loaded foam are:

0.25 DB FOAM		60 DB FOAM
1.16	$\epsilon_{_{\mathrm{S}}}$	41.0
1.01	$\epsilon_{\infty}$	1.6
0.6497 ns	$ au_0$	0.3450 ns
2.954E-04 S/m	σ	3.902E-01 S/m

The magnetic materials are duals of the above as described earlier with the relative conductivity scaled by the ratio  $\mu_0/\epsilon_0$ . The Nickel Ferrite parameters are  $\mu_{\rm e}=1$ ,  $\mu_{\rm s}=100$ ,  $\tau_0=20$  ns and  $\sigma^{\star}=0.0$ . For the 0.25 dB foams there are 10 cells per wavelength at approximately 3.0 GHz. For the 60 dB foam there are 10 cells per wavelength at approximately 2.35 GHz, as the 60 dB foam has a higher dielectric constant.

Figures 2-10 show the real and imaginary parts of the 0.25 dB and 60 dB foam permeability, the real and imaginary parts of

the Nickel Ferrite permeability, and the magnetic susceptibilities versus time for all three materials.

Figures 11-14 show the co-polarized far zone electric field versus time and the co-polarized RCS for the 0.25 dB and 60 dB dielectric foam spheres respectively.

Figures 15-18 show the co-polarized far zone electric field versus time and the co-polarized RCS for the 0.25 dB and 60 dB mangetic foam spheres respectively.

Figures 19-20 show the co-polarized far zone electric field versus time and the co-polarized RCS for the default Nickel Ferrite sphere using the dispersive FDTD method and the non-dispersive FDTD method. For the non-dispersive method, an equivalent permeability and magnetic conductivity were defined at 30 MHz from (16) with  $\mu$  replacing  $\epsilon$  as

$$\mu_{\Gamma} = \mu' |_{(\omega = 2\pi 30E6)}, \quad \sigma^* = \omega \mu'' \mu_0 |_{(\omega = 2\pi 30E6)}$$

# XI. SAMPLE PROBLEM SETUP

The code as furnished models a 6.67 m radius Nickel Ferrite dispersive magnetic sphere and computes backscatter far zone scattered fields at angles of  $\theta$ =22.5 and  $\phi$ =22.5 degrees. The corresponding output data files are also provided, along with a code to compute Radar Cross Section using these data files. In order to change the code to a new problem, many different parameters need to be modified. A sample problem setup will now be discussed.

Suppose that the problem to be studied is RCS backscatter versus frequency from a 28 cm by 31 cm perfectly conducting plate with a 3 cm dielectric coating with a dielectric constant of  $4\epsilon_0$  using a  $\theta$ -polarized field. The backscatter angles are  $\theta$ =30.0 and  $\phi$ =60.0 degrees and the frequency range is up to 3 Ghz.

Since the frequency range is up to 3 Ghz, the cell size must be chosen appropriately to resolve the field IN ANY MATERIAL at the highest frequency of interest. A general rule is that the cell size should be 1/10 of the wavelength at the highest frequency of interest. For difficult geometries, 1/20 of a wavelength may be necessary. The free space wavelength at 3 GHz is  $\lambda_0$ =10 cm and the wavelength in the dielectric coating at 3 GHz is 5 cm. The cell size is chosen as 1 cm, which provides a resolution of 5 cells/ $\lambda$  in the dielectric coating and 10 cells/ $\lambda_0$  in free space. Numerical studies have shown that choosing the cell size  $\leq$  1/4 of the shortest wavelength in any material is the practical lower limit. Thus the cell size of 1 cm is barely adequate. The cell size in the x, y and z directions is set in the common file through variables DELX, DELY and DELZ. Next the problem space size must be large enough to accomodate the

scattering object, plus at least a five cell boundary (10 cells is more appropriate) on every side of the object to allow for the far zone field integration surface. It is advisable for plate scattering to have the plate centered in the x and y directions of the problem space in order to reduce the cross-polarized backscatter and to position the plate low in the z direction to allow strong specular reflections multiple encounters with the A 10 cell border is chosen, and the problem space size is chosen as 49 by 52 by 49 cells in the x, y and z directions As an initial estimate, allow 2048 time steps so respectively. that energy trapped within the dielectric layer will radiate. Thus parameters NX, NY and NZ in COMMOND. FOR would be changed to reflect the new problem space size, and parameter NSTOP is changed to 2048. If all transients have not been dissipated after 2048 time steps, then NSTOP will have to be increased. Truncating the time record before all transients have dissipated will corrupt frequency domain results. Parameter NZFZ must be equal to 1 since we are interested in far zone fields only. Parameter MAGNET must be equal to 0 for the dielectric scatterer. To build the object, the following lines are inserted into the BUILD subroutine:

```
C
С
      BUILD THE DIELECTRIC SLAB FIRST
      ISTART=11
      JSTART=11
      KSTART=11
      NXWIDE=28
      NYWIDE=31
      NZWIDE=3
      MTYPE=2
      CALL DCUBE(ISTART, JSTART, KSTART, NXWIDE, NYWIDE, NZWIDE, MTYPE)
C
С
      BUILD PEC PLATE NEXT
C
      ISTART=11
      JSTART=11
      KSTART=11
      NXWIDE=28
      NYWIDE=31
      NZWIDE=0
      MTYPE=1
      CALL DCUBE(ISTART, JSTART, KSTART, NXWIDE, NYWIDE, NZWIDE, MTYPE)
```

The PEC plate is built last on the bottom of the dielectric slab to avoid any air gaps between the dielectric material and the PEC plate. In the common file, the incidence angles THINC and PHINC have to be changed to 30.0 and 60.0 respectively, the cell sizes (DELX, DELY, DELZ) are set to 0.01, and the polarization is set to ETHINC=1.0 and EPHINC=0.0 for  $\theta$ -polarized fields. Since dielectric material 2 is being used for the dielectric coating, the constitutive parameters EPS(2) and

SIGMA(2) are set to  $4\epsilon_0$  and 0.0 respectively, in subroutine SETUP. This completes the code modifications for the sample problem.

# XII. NEW PROBLEM CHECKLIST

This checklist provides a quick reference to determine if all parameters have been defined properly for a given scattering problem. A reminder when defining quantities within the code: use MKS units and specify all angles in degrees.

### COMMOND. FOR:

- Is the problem space sized correctly? (NX, NY, NZ)
- 2) For near zone fields, is the number of sample points correct? (NTEST)
- 3) Is parameter NZFZ defined correctly for desired field outputs?
- 4) Is parameter MAGNET defined correctly for the type of scatterer?
- 5) Is the number of dispersive dielectric (NEDISP) and dispersive magnetic (NHDISP) materials defined correctly?
- 6) Is the number of time steps correct? (NSTOP)
- 7) Are the cell dimensions (DELX, DELY, DELZ) defined correctly?
- 8) Are the incidence angles (THINC, PHINC) defined correctly?
- 9) Is the polarization of the incident wave defined correctly (ETHINC, EPHINC)?
- 10) For other than backscatter far zone field computations, are the scattering angles set correctly? (THETFZ, PHIFZ)

# SUBROUTINE BUILD:

1) Is the object completely and correctly specified?

### SUBROUTINE SETUP:

- 1) Are the constitutive parameters for each material specified correctly? (EPS, XMU, SIGMA, SIGMAC)
- 2) Are the constitutive parameters for each dispersive material defined correctly? (EPSSTA, EPSINF, RELAXT, RELSIG, XMUINF, XMUSTA, RELAXT, RELSIG)

# FUNCTIONS SOURCE and DSRCE:

1) If the smooth cosine pulse is not desired, is it commented out and the Gaussian pulse uncommented?

# SUBROUTINE DATSAV:

1) For near zone fields, are the sampled field types and spatial locations correct for each sampling point? (NTYPE, IOBS, JOBS, KOBS)

# XIII. REFERENCES

- [1] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media,"

  <u>IEEE Trans. Antennas Propagat.</u>, vol. AP-14, pp. 302-307, May 1966.
- [2] G. Mur, "Absorbing boundary conditions for the Finite-Difference approximation of the Time-Domain Electromagnetic-Field Equations," <u>IEEE Trans. Electromagn. Compat.</u>, vol. EMC-23, pp. 377-382, November 1981.
- [3] R. J. Luebbers et. al., "A Finite Difference Time-Domain Near Zone to Far Zone Transformation," <u>IEEE Trans. Antennas Propagat.</u>, vol. AP-39, no. 4, pp. 429-433, April 1991.
- [4] C. Balanis, <u>Advanced Engineering Electromagnetics</u>, New York: Wiley, 1990, pp. 83-84.
- [5] R. J. Luebbers et. al., "A frequency-dependent Finite-Difference Time-Domain formulation for dispersive materials," <u>IEEE Trans. Electromagn. Compat.</u>, vol. EMC-32, pp. 222-227, August 1990.
- [6] R. J. Luebbers et. al., "A frequency-dependent Finite-Difference Time-Domain formulation for transient propagation in plasma," <u>IEEE Trans. Antennas Propagat.</u>, vol. AP-39, pp. 429-433, April 1991.
- [7] R. Holland, L. Simpson, K. S. Kunz, "Finite-Difference Analysis of EMP Coupling to Lossy Dielectric Structures,"

  <u>IEEE Trans. Electromagn. Compat.</u>, vol. EMC-22, pp. 203-209,
  August 1980.

# XI. FIGURE TITLES

- Fig. 1 Standard three dimensional Yee cell showing placement of electric and magnetic fields.
- Fig. 2 Real part of relative permeability versus frequency for 0.25 dB magnetic foam.

- Fig. 3 Imaginary part of relative permeability versus frequency for 0.25 dB magnetic foam.
- Fig. 4 Relative magnetic susceptibility versus time for 0.25 dB magnetic foam.
- Fig. 5 Real part of relative permeability versus frequency for 60 dB magnetic foam.
- Fig. 6 Imaginary part of relative permeability versus frequency for 60 dB magnetic foam.
- Fig. 7 Relative magnetic susceptibility versus time for 60 dB magnetic foam.
- Fig. 8 Real part of relative permeability versus frequency for Nickel Ferrite with  $\mu_{\rm s}$  = 100,  $\mu_{\rm o}$  = 1,  $\tau_{\rm o}$  = 20 ns.
- Fig. 9 Imaginary part of relative permeability versus frequency for Nickel Ferrite with  $\mu_{\rm s}$  = 100,  $\mu_{\rm o}$  = 1,  $\tau_{\rm 0}$  = 20 ns.
- Fig. 10 Relative magnetic susceptibility versus time for Nickel Ferrite.
- Fig. 11 Co-polarized far zone scattered field versus time for 0.25 dB dielectric foam sphere with 20 cm radius.
- Fig. 12 Co-polarized Radar Cross Section versus frequency for 0.25 dB dielectric foam sphere with 20 cm radius.
- Fig. 13 Co-polarized far zone scattered field versus time for 0.25 dB magnetic foam sphere with 20 cm radius.
- Fig. 14 Co-polarized Radar Cross Section versus frequency for 0.25 dB magnetic foam sphere with 20 cm radius.
- Fig. 15 Co-polarized far zone scattered field versus time for 60 dB dielectric foam sphere with 20 cm radius.
- Fig. 16 Co-polarized Radar Cross Section versus frequency for 60 dB dielectric foam sphere with 20 cm radius.
- Fig. 17 Co-polarized far zone scattered field versus time for 60 dB magnetic foam sphere with 20 cm radius.
- Fig. 18 Co-polarized Radar Cross Section versus frequency for 60 dB magnetic foam sphere with 20 cm radius.

- Fig. 19 Co-polarized far zone scattered field versus time for Nickel Ferrite sphere with 6.67 m radius using both dispersive FDTD and non-dispersive FDTD.
- Fig. 20 Co-polarized Radar Cross Section versus frequency for Nickel Ferrite sphere with 6.67 m radius using both dispersive FDTD and non-dispersive FDTD.

